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FILM CONDENSATION IN A VERTICAL TUBE WITH A CLOSED TOP

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INTRODUCTION

SEBAN and Hodgson's analysis [1] for condensation in vertical tubes has been used to obtain solutions for the condensation in a tube, closed at the top. This situation is associated with the condenser portion of a vertical wickless heat pipe. Ho and Tien [2] have obtained some results for such a system and their appraisal was one motivation for this calculation. Another was a preliminary examination of the conditions in which flooding might occur in such a system.

Some of the quantities associated with the problem, already given in ref. [1], are repeated here for convenience. They differ slightly, since here the liquid velocity was taken as positive downward for the liquid flow, the downward direction being positive, and the upward vapor velocity was also taken as positive. The liquid Reynolds number is then

$$\frac{\Gamma}{\mu_L} = \frac{\bar{\delta}^3}{3} - N_2\frac{\bar{\delta}^2}{2}. \qquad (1)$$

This and subsequent equations, involved the non-dimensionalization

$$\bar{\delta} = \delta\left(\frac{g}{\nu_L^2}\right)^{1/3}, \quad \bar{u} = u/(\nu_L g)^{1/3}, \quad N_2 = \frac{\tau_\delta}{\rho_L g}\left(\frac{g}{\nu_L^2}\right)^{1/3}.$$

With the Nusselt assumption of a linear temperature distribution in the condensate layers, the local energy balance on the liquid then gives

$$\frac{d(\Gamma/\mu_L)}{d\bar{z}} = \frac{N_T}{\bar{\delta}}, \quad N_T = \left[\frac{c_L(T_v - T_w)}{h_{fg}}\left(\frac{\alpha}{\nu}\right)_L\right], \qquad (2)$$

or, if the heat flux is prescribed

$$\frac{d(\Gamma/\mu_L)}{d\bar{z}} = \frac{q_0}{\mu_L h_{fg}}\left(\frac{\nu_L^2}{g}\right)^{1/3} \equiv N_H. \qquad (3)$$

The function, τ_δ, contained in the quantity N_2 is specified as for vapor flow in a smooth tube, with an augmentation factor according to the specification of Henstock and Hanratty [3] whereby the smooth tube function factor was increased by the factor $(1+1400F)$. For turbulent vapor and laminar liquid flows

$$F = \frac{\sqrt{2}\sqrt{(\Gamma/\mu_L)}}{Re_v^{0.9}}\left(\frac{\nu_L}{\nu_v}\right)\left(\frac{\rho_L}{\rho_v}\right)^{1/2}\left[1-\exp\left(-\frac{N_2}{\bar{\delta}}\right)\right]. \qquad (4)$$

No specification is available for laminar vapor flow. Equation (4) was used for laminar vapor flow, for which conditions the value of $(1+1400F)$ was in any case nearly unity. The singular nature of F for $Re_v \to 0$ did not affect the calculations, which were initiated by neglecting the interfacial shear in a small length, where the condensate layer began at the top of the tube. The shear, τ_δ, calculated from the corrected friction factor, was further increased by a term accounting approximately for the mass transfer effect. As in ref. [1] the group N_2 becomes

$$N_2 = \left(\frac{c_f}{2}\right)_c \frac{\rho_v}{\rho_L}(\bar{u}_v + \bar{u}_{L\delta})^2 + \frac{N_T}{\bar{\delta}}(\bar{u}_v + \bar{u}_{L\delta}). \qquad (5)$$

If the heat flux is specified the coefficient of the last term is N_H instead of $N_T/\bar{\delta}$.

In addition to the above relations, there is the equality of mass rates of liquid and vapor flow, $\dot{m}_v = \dot{m}_L$, at any location, $\bar{z}$. In terms of Reynolds numbers this is

$$\frac{4\mu_L}{\mu_v}\frac{\Gamma}{\mu_L} = \frac{4\dot{m}_v}{\pi D\mu_v}. \qquad (6)$$

The numerical solution begins at the top and proceeds downward; the execution is described in ref. [1]. It specifies the liquid Reynolds number and hence $\bar{\delta}$ as a function of $\bar{z}$. With $\bar{\delta}$ known the Nusselt number, $h(\nu^2/g)_L^{1/3}/k_L = 1/\bar{\delta}$ can be evaluated. The increase in $\bar{\delta}$ with $\bar{z}$ continues until $d(\Gamma/\mu_L)/d\bar{\delta} = 0$; this gives the maximum condensate flow rate, a situation that is associated with flooding, or 'hold up'. For comparison the flooding limit as proposed by Wallis is evaluated [4]

$$j_L^{*} + j_v^{*} = c_w; \quad j_i^{*} = j_i\left(\frac{\rho_i}{Dg(\rho_L - \rho_v)}\right)^{1/2}; \quad 0.7 < c_w < 1.0,$$

with $\rho_L \gg \rho_v$, and since $\dot{m}_v = \dot{m}_L$, this gives the limiting gas Reynolds number as

$$Re_{vL} \equiv \frac{4\dot{m}_v}{\pi D\mu_v} = \bar{D}^{3/2}\left(\frac{\nu_L}{\nu_v}\right)\left(\frac{\rho_L}{\rho_v}\right)^{1/2} c_w^2. \qquad (7)$$

Here c_w was taken as 0.70, for $c_w = 1.0$ the limiting Reynolds number becomes twice as large.

It is of interest to note that an analytical solution can be obtained if the friction coefficient is taken as zero, so that N_2 is then specified by the last term of equation (5). With this specification the liquid Reynolds number is

$$\frac{\Gamma}{\mu_L} = \frac{A\bar{\delta}^3}{1+B\bar{\delta}}; \quad A = \frac{1}{3} - \frac{1}{4}\left(\frac{N_T}{1+N_T}\right)\Bigg],$$

$$B = \left[\frac{2N_T}{(1+N_T)}\left(\frac{\rho_L}{\rho_v}\right)\frac{1}{\bar{D}}\right]. \qquad (8)$$

Substitution into equation (2) and integration gives

$$\frac{A\bar{\delta}^4}{1+B\bar{\delta}} - \frac{A}{B^4}\left[\frac{(\bar{\delta}B)^3}{3} - \frac{(\bar{\delta}B)^2}{2} + \frac{(\bar{\delta}B)}{1}\right] - \log(1+B\bar{\delta}) = N_T\bar{z}. \qquad (9)$$

This can be approximated empirically as

$$\bar{\delta} = 1.36\left(\frac{N_T\bar{z}}{A}\right)^{0.295}(B)^{0.18}. \qquad (10)$$

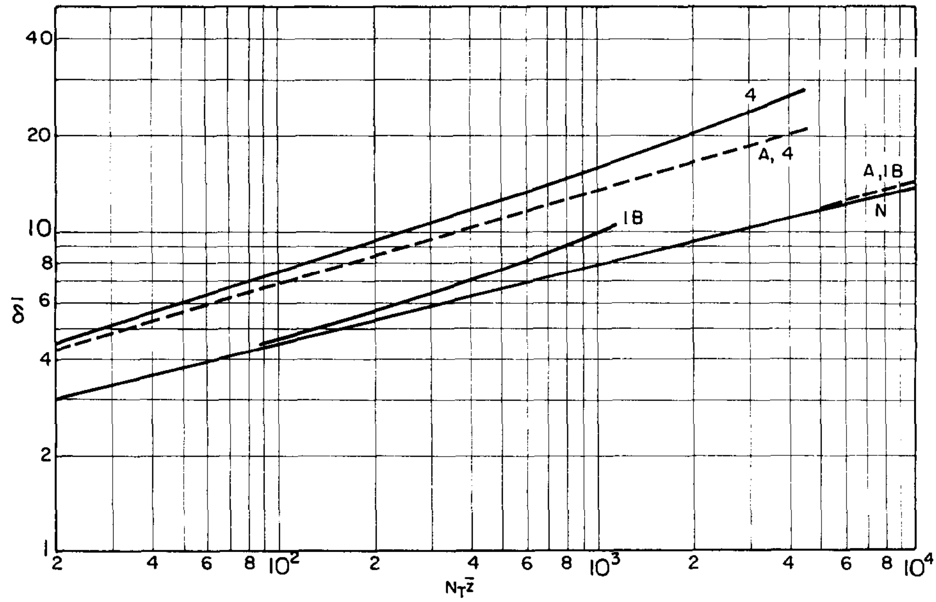

FIG. 1. Liquid layer thicknesses for Case 1B and 4. The solid curves are the numerical solutions. Curves A are from equation (10). Curve N is the Nusselt solution.

RESULTS WITHOUT A FLOODING LIMIT

Six cases were calculated for conditions associated with the 2.42 cm I.D., 30.5 cm high tube that was used by Ho and Tien [2] to condense methanol by cooling with an external water flow through an external annular section, with saturated vapor at a temperature of 63°C, and a wall temperature about 25°C below this temperature. Vapor properties were specified, and liquid properties were taken at the average of the vapor and the wall temperatures. These properties are shown in Table 1. Cases 1–3, were calculated for three of the operating conditions based on a constant wall temperature, equal to the temperature of one wall measured near the bottom of the condensing section. Because of the uncertainty about the actual wall temperature variation, Cases 1A, 2A and 3A were calculated for a constant heat flux, which is the average flux as deduced from an energy balance on the cooling water flow, the magnitude used in the evaluation of the experimental heat transfer coefficient.

Table 1 gives the hydrodynamic results. Columns 2–4 are associated with the initial and specified conditions, the temperature difference, the value of N_T [equation (2)], and the value of D. For Cases A, for constant flux, Column 2 gives the flux (kW m^{-2}), and Column 3 the value of N_H [equation (3)]. The remaining columns give quantities associated with the film near the end of the calculation, at the value of $\bar{z}$ indicated in Column 5. There are given successively the final value of $\bar{\delta}$, the value of δ/D corresponding to this, the liquid Reynolds number, the gas Reynolds number based on $D' = D-2\delta$ and on D, the value of N_2, and in Columns 13 and 14, the values of 1400F and of CMI, which denotes the second term of equation (5). Column 12 gives the limiting (flooding) gas Reynolds number from equation (7). This depends only upon D and on its properties, it is 15 900 for all these cases. For all cases, Cases 1–3A, the contribution of term F is small, and the term CMI is the important element of N_2. The gas Reynolds numbers are far below the flooding limit.

Table 2 gives the average Nusselt number, $\bar{h}(v_L^2/g)^{1/3}/k_L$, for these cases, with P indicating the predicted value. For comparison, the values for the Nusselt solution for a vertical plate with a stagnant vapor are indicated by N. The experimental values are given in the column headed by the letter E. For all cases, Cases 1–3A, the predicted Nusselt number, evaluated for a $\bar{z}$ of about 10^4, corresponding to the 30.5 cm length, is only slightly less than the values specified by the Nusselt solution. This means of course, that the final film thickness as predicted is only very slightly larger than that specified by the Nusselt solution. The influence of the interfacial shear is very small in these cases. The experimental coefficients, evaluated from the average flux and a temperature difference near the bottom of the condenser, are of the order of the prediction. They should be, because for the small liquid Reynolds numbers that are involved there would be little augmentation of the transfer due to the effect of waves.

To investigate the effect of an increase in the condenser length, the calculation was extended to $\bar{z} = 10^5$ ($z = 305$ cm) as Case 1B, using the temperature differences associated with Case 1. Case 1B therefore included Case 1. Thus the final value of $\bar{\delta}$ was increased to 10.4; with $\delta/D = 0.013$; the film was still thin enough for curvature to be negligible. But the final gas Reynolds number was greater than the flooding value associated with $c_w = 0.70$; but that value would be twice as large for $c_w = 1.0$. Figure 1 shows the value of $\bar{\delta}$ as a function of $N_T\bar{z}$, the final value being 1140. For Case 1 the final value is about 114 and the comparison of δ with the value indicated for the Nusselt solution [equation (5)] indicates the small effect of friction up to the value of $N_T z = 114$. The effect increases for larger values, as the gas velocity increases, and $\bar{\delta}$ is 1.3 times the Nusselt value at the end of the calculation in Case 1B. As a further reference, Curve A shows the indication of equation (10) to show, as does Column 14 of Table 1, the small influence of the friction term in equation (5).

In a further search for a case which would exhibit a limiting condition, the temperature difference was increased to 1000°C for Case 4, but the calculation was terminated at $\bar{z} = 10^4$. This produced a final $\bar{\delta}$ of 28.5, with δ/D thus being 0.036, and a vapor Reynolds number of 36 400. No indication of flooding was obtained. Figure 1 shows the variation of $\bar{\delta}$ which at the end of the calculation attained a value 2.5 times that indicated by the Nusselt solution. Table 2 indicates the associated reduction in the Nusselt number. In this case the temperature difference is so large (N_T large) that the second term of equation (5) dominates, as indicated by Columns 11 and 14 of Table 1. Consequently, the prediction from equation (10) is in the regions of the calculated values of $\bar{\delta}$, as shown by Curve A of Fig. 1. The values of $\bar{\delta}$ progressively exceed that specification as

Table 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Case	$(T_v - T_w)$°C	$N_T \times 10^2$	$\bar{D}$	$\bar{z}$	$\bar{\delta}$	$\frac{\Gamma}{\mu_L}$	$\frac{\delta}{D} \times 10^3$	Re'_v	Re_v	N_2	Re_{vL}	$1400F$	CMI
1	25.2	1.14	801	9450	4.72	30.7	5.9	4610	4450	0.40	15 900	0.192	0.28
2	26.2	1.19	801	9450	4.79	31.7	5.9	4770	4710	0.43	15 900	0.199	0.30
3	29.2	1.34	801	9450	4.96	34.5	6.2	5190	5110	0.50	15 900	0.217	0.36
1A	(46.6)*	0.31	801	9810	4.79	31.0	46.0	4650	4600	0.49	15 900	0.191	0.37
2A	(53.9)*	0.40	801	9700	5.30	38.7	6.6	5830	5750	0.77	15 900	0.238	0.59
3A	(60.6)*	0.41	801	9630	5.35	39.5	6.7	5950	5850	0.80	15 900	0.243	0.62
1A	1000	41.5	801	9670	28.5	245	36	39 200	36 400	18.4	15 900	0.523	13.7
1B	25.2	1.14	801	92 700	10.4	156	13	23 800	23 200	4.0	15 900	1.14	0.64
5	25.2	1.14	159	7790	11.8	19.2	74	3350	2840	7.5	1410	4.31	0.43
6	25.2	1.14	80	1280	9.2	3.3	115	636	495	6.1	505	9.02	0.22
7		1.14	37	183	3.6	1.26	97	23 300	18 800	2.2	15 900	17.3	0.45

Properties and parameters

ρ_L (kg m^{-3})	ρ_v	ν_L (m^2 s^{-1})	ν_v	h_{fg} (kJ kg^{-1})	$(\nu/\alpha)_L$	D (m)
765	1.10	0.52×10^{-6}	0.974×10^{-5}	1120	5.10	0.0242

for Cases 5 and 6 $D = 0.00484$ and 0.00242 m

for Case 7 $\nu_L = 0.52 \times 10^{-4}$, other conditions unchanged, including N_1, so that $\left(\frac{\nu}{\alpha}\right)_L = 510$ for $(T_v - T_w) = 2520$ or $\nu/\alpha = 5.10$ for $T_v - T_w = 25.2$

* For these cases this column is the assumed constant heat flux (kW m^{-2}) and N_T is N_H, equation (6).

Table 2

	Nussselt numbers											
	Constant temperature							Constant heat rate				
	Local			Average				Local		Average		
Case	P	N	P/N	P	N	P/N	E	P	N	P	N	Case
1	0.212	0.218		0.284	0.290		0.283	0.209	0.225	0.318	0.338	1A
2	0.210	0.218		0.280	0.290		0.342	0.189	0.204	0.293	0.306	2A
3	0.202	0.218		0.271	0.290		0.312	0.187	0.203	0.888	0.304	3A
1B	0.087	0.121	0.72	0.145	0.162	0.89						
4	0.035	0.088	0.40	0.056	0.117	0.49						
5	0.084	0.230	0.36	0.210	0.307	0.68						
6	0.108	0.360	0.30	0.226	0.480	0.47						
7	0.276	0.970	0.28	0.604	1.29	0.47						

friction becomes increasingly important. It appears that a continuation of Case 4 to large values of $\bar{z}$ would have ultimately produced a limiting situation.

RESULTS WITH A FLOODING LIMIT

In the search for a limit the conditions of Case 1 were retained except that the tube diameter was reduced, to 0.484 cm for Case 5 and to 0.242 cm for Case 6. Limits were attained, but rather imprecisely because the increment in $\bar{\delta}$, which depended on $\bar{\delta}$, becomes too large near the limit. In both cases the incrementing overshot the limit and caused a failure in the scheme intended for the termination of the calculation. The results, as will be shown, do indicate a limit in the Reynolds number that is relatively satisfactory. The values of $\bar{z}$, as given in Column 5 of Table 1, may be slightly low. At the limit the values of δ/D, 0.074 for Case 5 and 0.115 for Case 6, are large enough to cast doubt upon the neglect of curvature of the film that is part of the analysis.

Figure 2 shows, for these cases, the variation of $\bar{\delta}$ as a function of $\bar{z}$. The Nusselt values are shown, Curve N, and the values from equation (10) by Curves A. Because of the small temperature difference there is relatively little departure of Curves A from that of the Nusselt solution.

The results for Cases 5 and 6 are shown by points on Fig. 2. These are selected from the print of the results but for about the last four points these points are the printed results, each being at an interval of five increments in $\bar{\delta}$. At the true limit $d\bar{\delta}/d\bar{z}$ should become infinite. The final points do not show this but the approach to such a slope is clear.

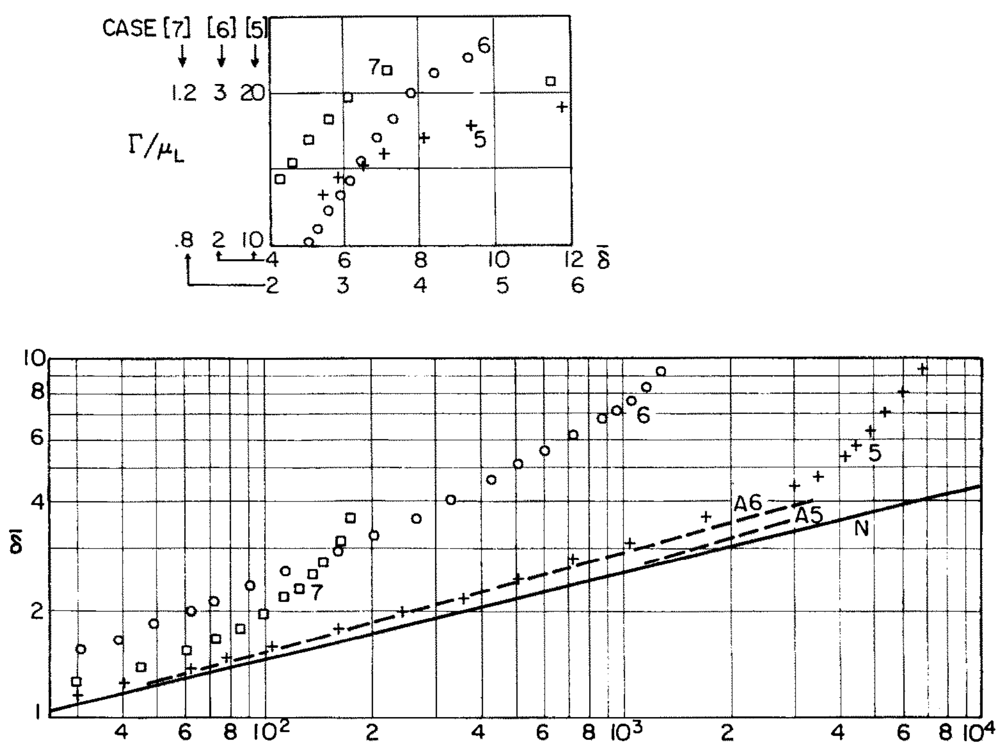

FIG. 2. Liquid layer thickness for Cases 5–7. The points are from the numerical solution. Curves A are from equation (10). Curve N is the Nusselt solution. The inset shows the liquid Reynolds number as a function of $\bar{\delta}$ at the flooding limit.

Figure 2 also shows the liquid Reynolds number, Γ/μ_L, as a function of $\bar{\delta}$ near the end of the calculation. The points are from the print, at intervals of five increments in $\bar{\delta}$. For Case 5, the condition $d(\Gamma/\mu_L)/d\bar{\delta} = 0$ is nearly realized at the last point, for $\Gamma/\mu_L = 3.3$. For this final condition, Columns 10 and 12 indicate that equation (7) does give the limit, within the factor of two associated with the choice of c_w in that equation. The gas Reynolds number in Case 6 is, however, so low as to make questionable the evaluation of F from equation (4), and the limit would not have been attained until a much larger value of z, and of the gas Reynolds number, if the dominant influence of F on the friction had been eliminated.

A final case, Case 7, is another situation in which a limit was attained. In it the conditions were those of Case 1, including the value of N_T, except that only the kinematic viscosity of the liquid was increased by a factor of 100. The increase in the viscosity of the liquid produced a limit at $\bar{z} = 183$ ($z = 4.9D$), with a gas Reynolds number of 18 800, of the order of the value of 15 900 from equation (7). This happens to be the same as in Case 1. Figure 2 shows the results and for them the limit $d\bar{\delta}/d\bar{z} \to \infty$ is approached more closely than it is for Cases 5 and 6. The representation of $\bar{\delta}$ as a function of $\bar{z}$ shows the last point for increasing Γ/μ_L at $\Gamma/\mu_L = 1.26$. But this does not indicate $d(\Gamma/\mu_L)/d\delta \to 0$.

CONCLUSIONS

The countercurrent flow problem of film condensation on the wall of a tube with a closed top end has been investigated for the case of a laminar film flow of the Nusselt type in which the curvature of the film has been neglected. The calculation of the film thickness and the Nusselt number for a particular situation associated with an experiment indicated that for the experiment the effect of interfacial shear on the film flow was very small and therefore the classical Nusselt solution was adequate for the prediction of results.

By diminishing the diameter of the tube or by altering the properties of the fluid, limiting situations were attained in the prediction of the film thickness, and the vapor Reynolds number for the limiting situation in some cases show a degree of agreement with an empirical specification of the gas Reynolds number at which flooding is expected.

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